Ash Lightfoot alightfoot@hse.ru

Statement of Teaching Philosophy

While experimental result drives scientific discovery, new mathematics often arises from connecting what we currently know. This makes mathematics instructors particularly privileged because learning is fundamentally about making connections. It is building connections to stimulate learning which guides me in structuring a course and my teaching.

Establishing a course syllabus that follows a logical sequence is the first opportunity to encourage connectivity. A collection of disjoint topics may be appropriate for an advanced graduate course but in an elementary course we should maximize our ability to motivate new concepts. Of course, this is easier to achieve if an instructor has free reign, but in most large undergraduate courses there is a fixed, sequential syllabus that must be adhered to; it is the instructor who must adapt. For example, in the *Finite Mathematics* course at Indiana University, linear algebra is covered after probability. Thus, when I first introduce matrices and matrix multiplication, I ask students to compute some probabilities using Markov chain theory (in a guided way) so they are immediately exposed to a connection between matrices and their earlier course work, as well as to the utility of these new tools.

On the other hand, given the opportunity to have a more flexible syllabus, I have, for example, used knot theory to motivate and introduce students to certain elementary linear algebra. Indeed, the problem of distinguishing knots (circles in 3-dimensional space) is readily grasped by students, and one simple approach to this problem is called knot coloring. This means that arcs of a picture of a knot are colored so that when two arcs cross, certain conditions are satisfied. Upon increasing the number of colors to work with and the complexity of the knot it becomes natural to introduce modulo arithmetic, matrices and determinants.

The lecture itself is the central place to stimulate student connections. I do so in my teaching by encouraging students to ask themselves "why?" as often as possible. Although there is no quick fix to turning shy students into hand-raisers, I can at least help students internally answer their questions by emphasizing the logic in a calculation. This is achieved by clearly defining each step and pausing often to face the class and review what has happened. In addition, I frequently ask the audience to predict what step might come next. The latter technique is particularly useful at discouraging students from treating a lecture as an exercise in transcription, copying the instructor verbatim without critically assessing the content. Pausing and putting the onus on students may, for instance, remind them that the blackboard work is using mathematics they were fluent with in high school. By encouraging students to be active participants of a lecture, I may even hope to have more students come up to me after class and say, "I found the answer this way instead. Is it right?".

A less direct mechanism to encourage students to ask questions during a lecture is to ensure that this is their primary source of information — if students can rely on a textbook then they are more likely to devalue the importance of understanding during a lecture, convincing themselves that they can always learn the material later. The lectures must therefore be clear and as self-contained as possible. (Anecdotally, I have noticed a distinct difference in attitude towards lectures in this respect between New Zealand, USA and Russian students; the latter are more inclined to request the instructor elaborate a point until it is well-understood.)

The end of each lecture should include some kind of debriefing: time allocated to summarize the new material and connect it to previous lectures. This may be a continuation of the lecture proper, or, more preferably, some form of student engagement so they can form their own connections. For example, at the end of a lecture in *Precalculus with Trigonometry* in which the basic trigonometric functions were introduced, I would have the class take an open-book quiz with a question such as "Sketch the graph of $e^{\cos x}$." This not only requires students to contrast the behavior of two functions they have recently learned about, but it reviews the earlier concept of function composition.

My role with students is as a facilitator of learning, rather than a transmitter of information. As instructor of *Preparation for the Mathematics GRE* I would have students present their solutions on the blackboard, which spurred lively discussion amongst the (relatively small) class. This exposed students to multiple approaches to problems and to new ways of thinking about mathematics they had previously pigeon-holed as belonging to one particular course of their seemingly distant past. The math circles I have taught (in Moscow, Russia, and in Bloomington, USA) have also given me excellent practise at facilitating learning, for these school students begin with a blank slate, as it were, and lack the credulity that years of drilled instruction is apt to produce. Consequently, mathematics must flow in the direction they think logical, which may differ from that of a standard college course. For example, during a lesson when graph theory was introduced, it was remarked that the complete graph on five vertices could not be embedded in the plane. While a college instructor may choose not to explore the point and promptly move on without apparent objection from the audience, this particular audience insisted we prove the assertion (and, indeed, this inspired the next week's lesson).

Finally, while its beauty and intrigue is more than sufficient to motivate those of us who study mathematics, we should take every opportunity to connect the subject to the world in which our students live. Now, asking a *Calculus* student to maximize the area of a field to be fenced off by a farmer is hardly a legitimate real-world application of optimization (farmers have more pressing matters than calculus when planning their fences). However, finding the maximum current to arise in an electrical circuit *is* a problem a physics or engineering student will encounter in their future studies.

My approach to teaching mathematics can thus be summarized as a perpetual adaptation to help students see the subject as a connected, cohesive whole, and as something of genuine purpose worth studying. I have demonstrated the ability to do so in a variety of environments, and strive to become more flexible as I continue to teach and expose myself to new settings and student needs.